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MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Some Problems of Orbital Prediction

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Phil. Trans. R. Soc. Lond. A 1967 **262**, 100-105
doi: 10.1098/rsta.1967.0035

Some problems of orbital prediction

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Attention is paid to the problem of predicting accurately (*a*) the orbital elements of a balloon satellite over long periods of time, and (*b*) the time the satellite will pass through a given point in the orbit, a few days or weeks in advance of the event.

It is shown that by computing the solar radiation pressure and gravitational perturbations of the orbit of the balloon satellite Echo 2, the orbital elements can be predicted reasonably accurately several months in advance. For the balloon satellite Explorer 19, allowing for air drag at perigee, computed from a simple atmospheric model in addition to the effect of solar radiation pressure, results in significant improvement in predicting the period of revolution a few months in advance.

Finally, by numerically integrating the air-drag effect round the orbit, it is shown that a considerable improvement can be made in the accuracy of predicting the time at which a satellite will pass through a given point on the orbit.

1. Introduction

In order to understand the evolution of a satellite orbit and to assess the suitability of a satellite and its orbit for a particular research project, it is often necessary to predict the changes that will take place in the orbital parameters several months or even years in advance. For a satellite with a moderate or large perigee height and whose area/mass ratio is small, the Earth's gravitational field largely determines the changes in its orbit and, since this field is known quite accurately, long-term prediction of the orbital elements of the orbit is not at all difficult. For other satellites, however, the perturbations due to air drag and solar radiation pressure become important. The relative importance of these two effects depends on the height of the satellite, but below about 1000 km air drag is normally the more important and above this height solar radiation pressure is dominant. Some attempts to predict on a long-term basis the effects of these perturbations on the orbital elements of balloon satellites have been carried out at R.S.R.S. in the last few years and results for two such satellites are described in §2.

In §2 the effects of air drag have been calculated making the assumption (King-Hele 1964) that these arise as the result of an 'impulse' in the region of perigee. In §3 the effects of air drag occurring round all or part of the orbit are estimated and applied to a number of satellites for which the influence of solar radiation pressure is small compared with that of air drag. The results of this section are relevant to the practical case of short-term prediction—say, over a fortnight—of the position of satellites.

2. Long-term prediction of orbital elements

A few months after the launch of the balloon satellite Echo 2 (1964–04A) an attempt was made to predict the day-by-day variation of its orbital elements for the first 2 years of its life, using N.A.S.A. orbital elements for the day of launch as the starting-point. In the prediction calculations, allowance was made for the effects of solar radiation pressure and of the harmonics of the Earth's gravitational field, but the rate of change of period of revolution due to air drag was assumed constant. Figures 1 to 3 show the observed and predicted values of the argument of perigee, eccentricity and nodal period for Echo 2 during the first 600 days in orbit. The initial apogee and perigee heights were 1316 and 1029 km and the inclination 81·5°.

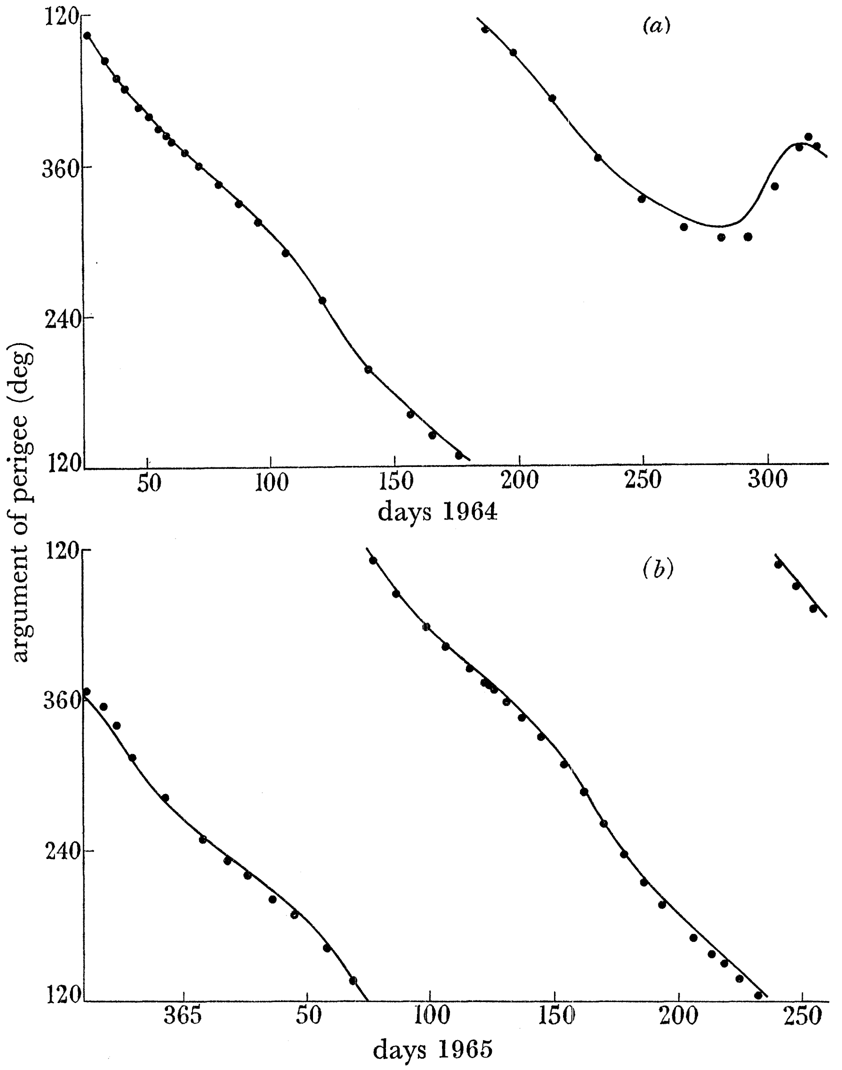

FIGURE 1. Echo 2: variation of argument of perigee with time.
——, Predicted; ●, observed.

The computed argument of perigee, shown in figure 1, is in good agreement with the subsequently observed variation of this element except near day 290 of 1964, when the solar radiation pressure perturbation and that due to the odd harmonics in the gravitational field became extremely large. At this time the rate of change of argument of perigee due to solar radiation pressure was over 6°/day and in the opposite direction to the motion of perigee due to the Earth's oblateness.

The computed eccentricity, shown in figure 2, also shows quite good agreement with the observed value for most of the time. The largest discrepancies occur in the first half of 1965 and there is also some indication during this period of a shift in phase of the variation. However, the agreement that has been achieved is probably sufficient for most purposes.

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The computed values of the nodal period, shown in figure 3, are not in as good agreement with the observed values as are the other elements. This is almost certainly due to the assumption that the air drag perturbation was constant throughout the period and may also be one of the reasons why the computed eccentricity values do not entirely agree with the observed values, since there is a small perturbation of the eccentricity due to air drag which has been neglected.

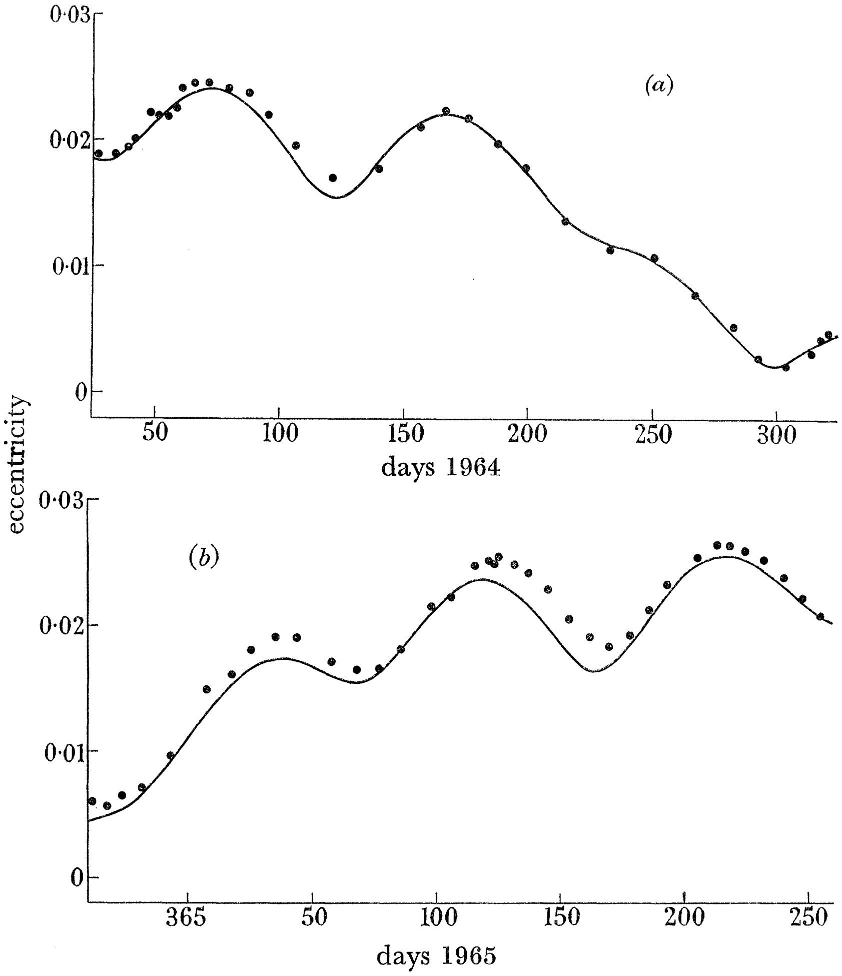

FIGURE 2. Echo 2: variation of eccentricity with time.
——, Predicted; ●, observed.

Among the other satellites for which the solar radiation pressure perturbations have been computed is Explorer 19 (1963–53A). This is a balloon satellite 12 ft. in diameter in an eccentric orbit with a perigee height of about 600 km, and air drag at this height introduces the major perturbation. In order to improve the accuracy with which the period of revolution can be predicted, a variable rate of change of period of the form shown in equation (1) was introduced to account for air drag:

$$\dot{T} = \dot{T}_0(1+F\cos\phi)\exp\{-(h-h_0)/H\}, \tag{1}$$

where $\dot{T}$ is the rate of change of period, $\dot{T}_0$ is a constant, related to the initial rate of change of period, F is the amplitude of the fractional diurnal variation of air density, ϕ is the angle between the perigee and the centre of the diurnal bulge of the atmosphere,

h is the perigee height, h_0 is the perigee height at the epoch of the elements, and H is the scale height in the region of the perigee. The term $(1+F\cos\phi)$ is a simple representation of the diurnal variation in the density at a fixed height above the Earth's surface and the exponential term represents the variation in air density with height.

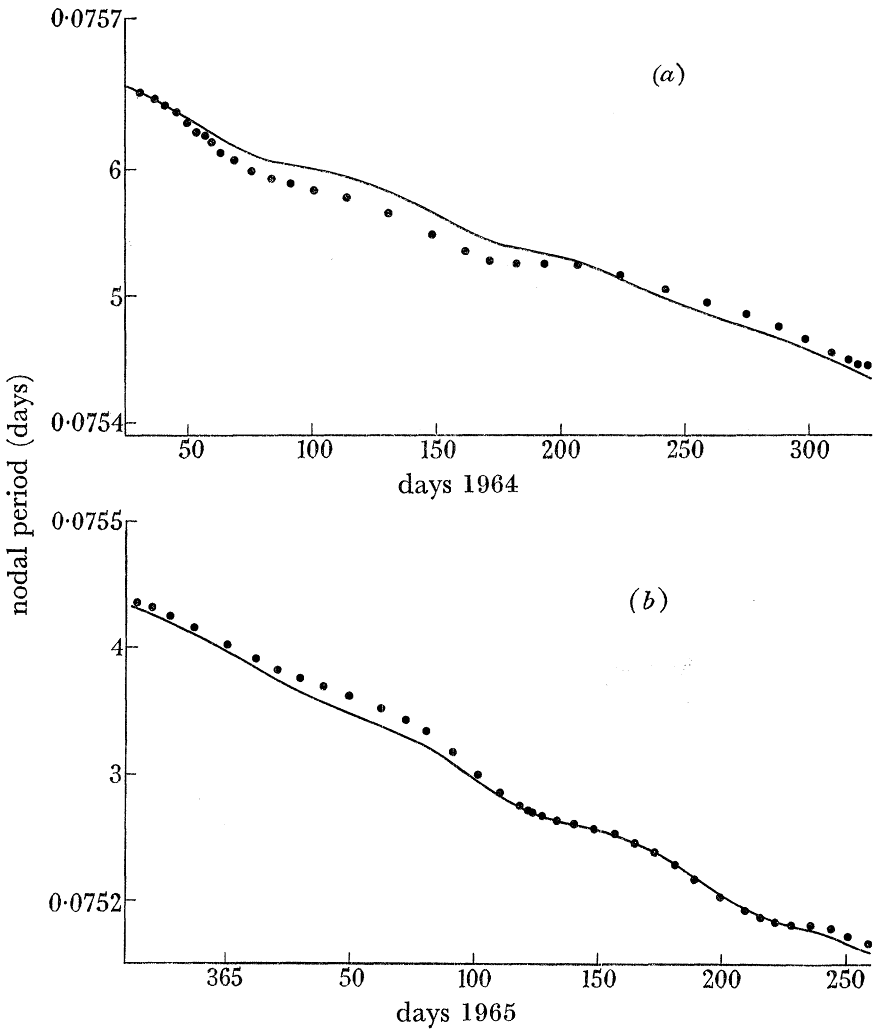

FIGURE 3. Echo 2: variation of nodal period with time. ——, Predicted; ●, observed.

As the position of perigee changes with respect to the diurnal bulge the $(1+F\cos\phi)$ term modifies the rate of change of period accordingly, and if the height of perigee above the Earth's surface changes, the value of T is modified by the exponential term. Equation (1) can only be applied to orbits of large eccentricity when it can be assumed that nearly all the air drag takes place at perigee.

Equation (1) has recently been applied to the prediction of the nodal period of Explorer 19, and figure 4 shows the results of this first attempt. The dotted curve shows the prediction on the basis of a constant air drag, the continuous curve shows the prediction by means of equation (1) and the small circles are observed values of the period. Although no particular effort was made to obtain accurate values of F and H for the analysis it is clear from figure 4 that the introduction of the varying drag term has made a considerable improvement in the accuracy of the prediction. The large departure of this curve from the observed values after day 235 is probably due to the large solar flares that occurred around this time.

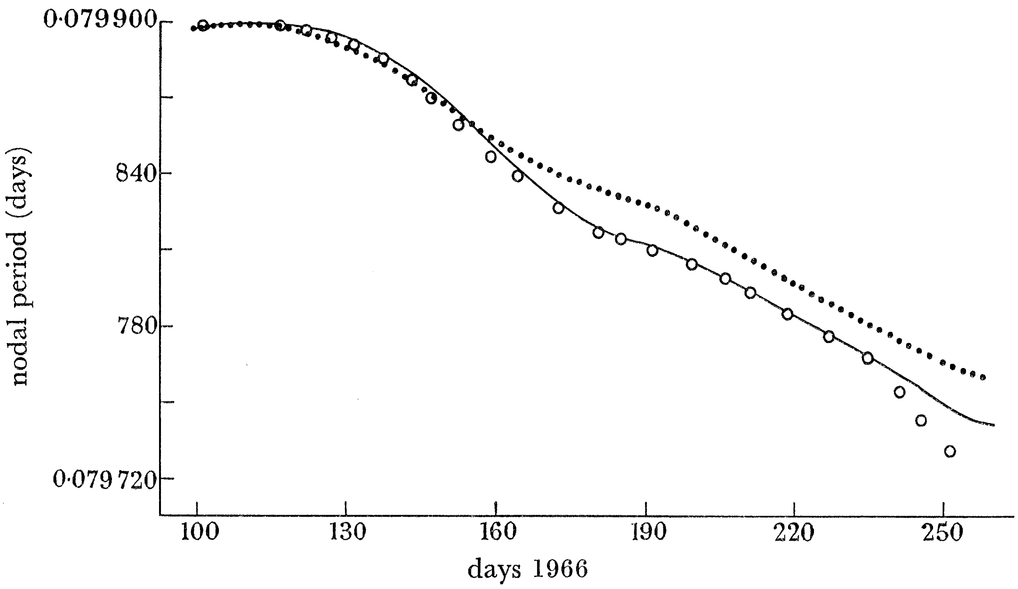

FIGURE 4. Explorer 19: variation of nodal period with time. . . ., Predicted with s.r.p. and constant drag; ——, predicted with s.r.p. and varying drag; ○, observed nodal period.

3. Short-term prediction of the time at which a satellite reaches a given position in its orbit

For a small-eccentricity orbit equation (1) is insufficiently accurate to determine the variations in the rate of change of period, because air drag takes place round most of the orbit and not just at perigee. If, however, equation (1) were integrated around the orbit a realistic value of $\dot{T}$ during the revolution might be obtained. Thus,

$$\dot{T} = \frac{\dot{T}_0}{2\pi} \int_0^{2\pi} (1 + F \cos \phi_u) \exp\left\{-\frac{h_u - h_0}{H}\right\} du, \tag{2}$$

where ϕ_u is the angle between the satellite and the centre of the diurnal bulge, h_u is the height of the satellite, and u is the argument of latitude of the satellite.

Equation (2) has been tried experimentally on several satellites for periods of about 2 weeks, and table 1 indicates some of the results obtained.

Two of the satellites shown in table 1 are in eccentric orbits with apogee heights above 2000 km and, as has already been mentioned, at these heights the effects of solar radiation pressure are more important than those of air drag. The perturbations due to solar radiation pressure are, however, considerably smaller than those of air drag near the perigees of the orbits, which are at a height of about 200 km, and have, consequently, been ignored.

Table 1 shows that the accuracy of predicting the times at which the satellites pass through a given position on their respective orbits is improved by using equation (2).

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It is not possible to say whether the improvement shown is typical, because only a few experimental tests have been carried out so far. However, the results are encouraging.

Table 1. Accuracy of prediction

satellite	period of prediction (days)	error in prediction at end of period (min): constant T	error in prediction at end of period (min): $\dot{T}$ given by equation (2)
1960 γ2	16	−7·4	−2·8
1962 $\beta\tau$6	14	+2·40	+0·66
1963-09A	12	−2·57	−0·31
1966-51A	15	−19·9	−2·0

4. Conclusions

It has been shown in §§2 and 3 that, by allowing for the calculated variation of air drag and solar radiation pressure on a satellite, an improvement can be made in the accuracy with which the orbital elements may be predicted and in the time at which the satellite reaches a given position in the orbit.

In calculating the air drag effect, no particular effort was made to use the best available values for the scale height or the amplitude of the diurnal variation in density, and yet a considerable improvement in the predictions was obtained. As more detailed information becomes available about the variation with height and solar cycle of these parameters of the atmosphere, a further gradual improvement can be expected in prediction accuracy within the limits of the atmospheric model that has been assumed.

The work described was carried out at the Radio and Space Research Station of the Science Research Council and is published with the permission of the Director.

Reference (Smith & Abayaratna)

King-Hele, D. G. 1964 *Theory of satellite orbits in an atmosphere*, p. 6. Butterworths Mathematical Texts.